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DO NOT WRITE ON THIS PACKET.

Do all work on a separate paper

Show work for every problem—
Even if it is a sentence telling what you did.

1–28 no calculator

29–45 yes calculator ← still show work.

BCI

Sample Examination I

Section I Part A

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given. Do not spend too much time on any one problem. Calculators may NOT be used on this part of the exam.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. If f is a continuous function defined by $f(x) = \begin{cases} x^2 + bx, & x \leq 5 \\ 5\sin\left(\frac{\pi}{2}x\right), & x > 5 \end{cases}$

then $b =$

(A) -6

(B) -5

(C) -4

(D) 4

(E) 5

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2. The graph of $y = 3x^2 - x^3$ has a relative maximum at

(A) $(0,0)$ only

(B) $(1,2)$ only

(C) $(2,4)$ only

(D) $(4,-16)$ only

(E) $(0,0)$ and $(2,4)$

3. A particle moves in the xy-plane so that its velocity vector at time t is $v(t) = (t^2, \sin(\pi t))$ and the particle's position vector at time $t = 0$ is $(1,0)$. What is the position vector of the particle when $t = 3$?

(A) $(9, \frac{1}{\pi})$

(B) $(10, \frac{2}{\pi})$

(C) $(6, -2\pi)$

(D) $(10, 2\pi)$

(E) $(10, 2)$

4. For what values of x does the curve $y^2 - x^3 - 15x^2 = 8$ have horizontal tangent lines?

(A) $x = -10$ only

(B) $x = 0$ only

(C) $x = 10$ only

(D) $x = 0$ and $x = -10$

(E) $x = -10$, $x = 0$, and $x = 10$

5. $\lim_{x\to\infty} \dfrac{10^8x^5 + 10^6x^4 + 10^4x^2}{10^9x^6 + 10^7x^5 + 10^5x^3} =$

(A) 0 (B) 1 (C) -1 (D) $\frac{1}{10}$ (E) $-\frac{1}{10}$

6. $\lim_{x\to 0} \dfrac{\int_1^{1+x} \frac{\cos t}{t}\, dt}{x}$ is

(A) $-\cos 1$ (B) $\cos 1$ (C) $-\sin 1$ (D) $\sin 1$ (E) nonexistent

7. If $f(x) = \sqrt{4\sin x + 2}$, then $f'(0) =$

(A) -2

(B) 0

(C) 1

(D) $\dfrac{\sqrt{2}}{2}$

(E) $\sqrt{2}$

8. If t is measured in hours and $f'(t)$ is measured in knots, then $\int_0^2 f'(t)\,dt =$ (Note: 1 knot = 1 nautical mile/hour)

(A) $f(2)$ knots

(B) $f(2) - f(0)$ knots

(C) $f(2)$ nautical miles

(D) $f(2) - f(0)$ nautical miles

(E) $f(2) - f(0)$ knots/ hour

9. An equation of the tangent line to the curve $x^2 + y^2 = 169$ at the point $(5, -12)$ is

(A) $5y - 12x = -120$

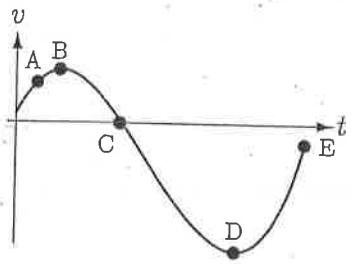
(B) $5x - 12y = 119$

(C) $5x - 12y = 169$

(D) $12x + 5y = 0$

(E) $12x + 5y = 169$

10.



The figure above shows the graph of the velocity of a moving object as a function of time. At which of the marked points is the *speed* the greatest?

(A) A

(B) B

(C) C

(D) D

(E) E

12. $\int \frac{1}{\sqrt{4-x^2}}\,dx =$

(A) $\text{Arcsin}\left(\frac{x}{2}\right)+C$

(B) $2\sqrt{4-x^2}+C$

(C) $\text{Arcsin}\,(x)+C$

(D) $\sqrt{4-x^2}+C$

(E) $\frac{1}{2}\text{Arcsin}\left(\frac{x}{2}\right)+C$

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13. If the graph of $f(x)=2x^2+\frac{k}{x}$ has a point of inflection at $x=-1$, then the value of k is

(A) 2

(B) 1

(C) 0

(D) -1

(E) -2

---

14. If $\int x\sec^2 x\,dx = f(x)+\ln|\cos x|+C$, then $f(x)=$

(A) $\tan x$ (B) $\frac{1}{2}x^2$ (C) $x\tan x$ (D) $x^2\tan x$ (E) $\tan^2 x$

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15. Which of the following is an equation of the line tangent to the curve with parametric equations $x=3e^{-t}$, $y=6e^{t}$ at the point where $t=0$?

(A) $2x+y-12=0$

(B) $-2x+y-12=0$

(C) $x-2y+9=0$

(D) $2x-y=0$

(E) $x+2y-15=0$

16. $\displaystyle\int \frac{dx}{2x^2+3x+1} =$

(A) $2\ln\left|\dfrac{2x+1}{x+1}\right| + C$

(B) $\ln\left|\dfrac{(2x+1)^2}{x+1}\right| + C$

(C) $\ln\left|\dfrac{x+1}{2x+1}\right| + C$

(D) $\ln\left|\dfrac{2x+1}{x+1}\right| + C$

(E) $\ln|(x+1)(2x+1)| + C$

---

17. If $x = \sin t$ and $y = \cos^2 t$, then $\dfrac{d^2y}{dx^2}$ at $t = \pi$ is

(A) -2 (B) $-\frac{1}{4}$ (C) 0 (D) $\frac{1}{4}$ (E) 2

---

18. If $y = x(\ln x)^2$, then $\dfrac{dy}{dx} =$

(A) $3(\ln x)^2$

(B) $(\ln x)(2x + \ln x)$

(C) $(\ln x)(2 + \ln x)$

(D) $(\ln x)(2 + x\ln x)$

(E) $(\ln x)(1 + \ln x)$

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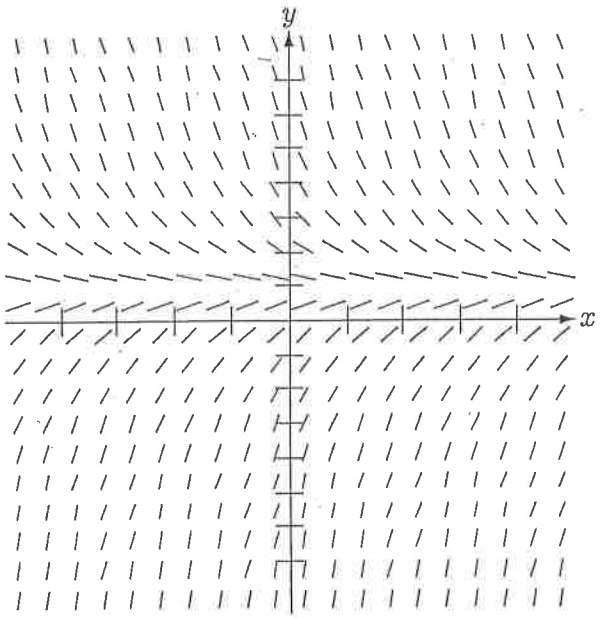
19. If $\displaystyle\int_0^6 \left(x^2 - 2x + 2\right) dx$ is approximated by a lower sum using three inscribed rectangles of equal width on the x–axis, then the approximation is

(A) 24 (B) 26 (C) 28

(D) 48 (E) 76

20.



Shown above is the slope field for which differential equation?

(A) $\frac{dy}{dx} = 1 - x$

(B) $\frac{dy}{dx} = x - y$

(C) $\frac{dy}{dx} = -\frac{x}{y}$

(D) $\frac{dy}{dx} = 1 + y^2$

(E) $\frac{dy}{dx} = 1 - y$

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23. If the length of a curve $y = f(x)$ from $x = a$ to $x = b$ is given by $L = \int_a^b \sqrt{e^{2x} + 2e^x + 2}\, dx$, then $f(x)$ is

(A) $2e^{2x} + 2e^x$

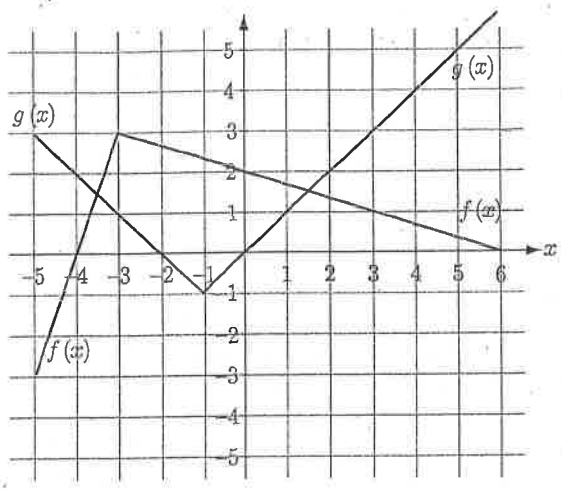
(B) $\frac{1}{2}e^{2x} + 2e^x + 2x$

(C) $e^x - x + 3$

(D) $e^x + 1$

(E) $e^x + x - 2$

24.



The functions f and g are piecewise linear functions whose graphs are shown above. If $h(x) = f(x)g(x)$, then $h'(3) =$

(A) $-\frac{8}{3}$ (B) $-\frac{1}{3}$ (C) 0

(D) $\frac{2}{3}$ (E) $\frac{8}{3}$

25. For which pair of functions $f(x)$ and $g(x)$ below, will the $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$?

	$f(x)$	$g(x)$
(A)	e^x	x^2
(B)	e^x	$\ln x$
(C)	$\ln x$	e^x
(D)	x	$\ln x$
(E)	3^x	2^x

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26. $\int_1^4 \frac{t-2}{(t+1)(t-4)}\,dt$ is found by using which of the following limits?

(A) $\lim_{b\to 2} \int_b^4 \frac{t-2}{(t+1)(t-4)}\,dt$

(B) $\lim_{b\to 1^+} \int_b^4 \frac{t-2}{(t+1)(t-4)}\,dt$

(C) $\lim_{b\to 4^-} \int_b^4 \frac{t-2}{(t+1)(t-4)}\,dt$

(D) $\lim_{b\to 1} \int_b^4 \frac{t-2}{(t+1)(t-4)}\,dt$

(E) $\lim_{b\to 4^-} \int_1^b \frac{t-2}{(t+1)(t-4)}\,dt$

27. The average value of the function $f(x) = \cos\left(\frac{1}{2}x\right)$ on the closed interval $[-4, 0]$ is

(A) $-\frac{1}{2}\sin(2)$

(B) $-\frac{1}{4}\sin(2)$

(C) $\frac{1}{2}\cos(2)$

(D) $\frac{1}{4}\sin(2)$

(E) $\frac{1}{2}\sin(2)$

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28. If n is a positive integer, then $\lim_{n\to\infty} \frac{1}{n}\left[\frac{1}{1+(1/n)} + \frac{1}{1+(2/n)} + \cdots + \frac{1}{1+(n/n)}\right]$ can be expressed as

(A) $\int_1^2 \frac{1}{x}\,dx$

(B) $\int_1^2 \frac{1}{x+1}\,dx$

(C) $\int_1^2 x\,dx$

(D) $\int_1^2 \frac{2}{x+1}\,dx$

(E) $\int_0^1 \frac{1}{x}\,dx$

Section I Part B

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given. Do not spend too much time on any one problem. A graphing calculator is required for some questions on this part of the examination.

In this test:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

29. The volume of the solid formed by revolving the region bounded by the graph of $y = (x-3)^2$ and the coordinate axes about the $x-$axis is given by which of the following integrals?

(A) $\pi \int_0^3 (x-3)^2 \, dx$

(B) $\pi \int_0^3 (x-3)^4 \, dx$

(C) $\pi \int_0^3 (x-3)^3 \, dx$

(D) $\pi \int_0^3 x(x-3)^2 \, dx$

(E) $\pi \int_0^3 x(x-3)^4 \, dx$

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30. Let f be the function given by $f(x) = \tan x$ and let g be the function given by $g(x) = x^2$. At what value of x in the interval $0 \leq x \leq \pi$ do the graphs of f and g have parallel tangent lines?

(A) 0 (B) 0.660 (C) 2.083

(D) 2.194 (E) 2.207

31. Let $f(t) = \frac{1}{t}$ for $t > 0$. For what value of t is $f'(t)$ equal to the average rate of change of f on the closed interval $[a, b]$?

(A) $-\sqrt{ab}$

(B) $\sqrt{ab}$

(C) $-\frac{1}{\sqrt{ab}}$

(D) $\frac{1}{\sqrt{ab}}$

(E) $\sqrt{\frac{1}{2}\left(\frac{1}{b} - \frac{1}{a}\right)}$

32. Let f be the function given by $f(x) = 3 + \int_0^x \cos(t^2)\, dt$. What is the smallest positive number a, for which $f'(a) = 0$?

(A) 1

(B) 1.253

(C) 1.571

(D) 1.772

(E) 3.142

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33. Let $R(t)$ represent the rate in gallons/hour at which water is leaking out of a tank, where t is measured in hours. Which of the following expressions represents the total gallons of water that leaks out in the first three hours?

(A) $R(3) - R(0)$

(B) $\dfrac{R(3) - R(0)}{3 - 0}$

(C) $\int_0^3 R(t)\, dt$

(D) $\int_0^3 R'(t)\, dt$

(E) $\frac{1}{3}\int_0^3 R(t)\, dt$

---

34. Which of the following gives the area of the region enclosed by the graph of the polar curve $r = 1 + \cos\theta$?

(A) $\int_0^{\pi} (1 + \cos^2\theta)\, d\theta$

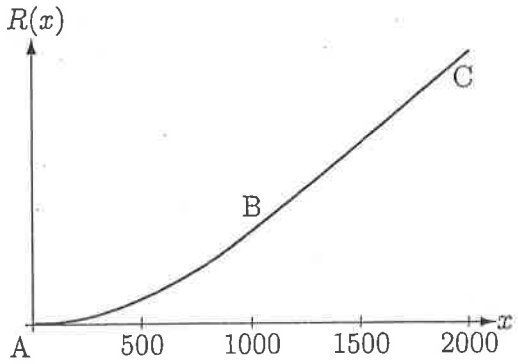
(B) $\int_0^{\pi} (1 + \cos\theta)^2\, d\theta$

(C) $\int_0^{2\pi} (1 + \cos\theta)\, d\theta$

(D) $\int_0^{2\pi} (1 + \cos\theta)^2\, d\theta$

(E) $\frac{1}{2}\int_0^{2\pi} (1 + \cos^2\theta)\, d\theta$

35.



The figure above shows a road running in the shape of a parabola from the bottom of a hill at A to point B. At B it changes to a line and continues on to C. The equation of the road is

$$R(x) = \begin{cases} ax^2, & \text{from A to B} \\ bx + c, & \text{from B to C} \end{cases}$$

B is 1000 feet horizontally from A and 100 feet higher. Since the road is smooth, $R'(x)$ is continuous. What is the value of b?

(A) 0.2

(B) 0.02

(C) 0.002

(D) 0.0002

(E) 0.00002

37.

x	−0.3	−0.2	−0.1	0	0.1	0.2	0.3
$f(x)$	2.018	2.008	2.002	2	2.002	2.008	2.018
$g(x)$	1	1	1	2	2	2	2
$h(x)$	1.971	1.987	1.997	undefined	1.997	1.987	1.971

The table above gives the values of three functions, f, g, and h near $x = 0$. Based on the values given, for which function does it appear that the limit as x approaches zero is 2?

(A) f only

(B) g only

(C) h only

(D) f and h only

(E) f, g, and h

38. If $\frac{dy}{dx} = \frac{x^2}{y}$ and $x = 1$ when $y = 1$, then $y =$

(A) $\frac{2}{3}x^2 + 12$

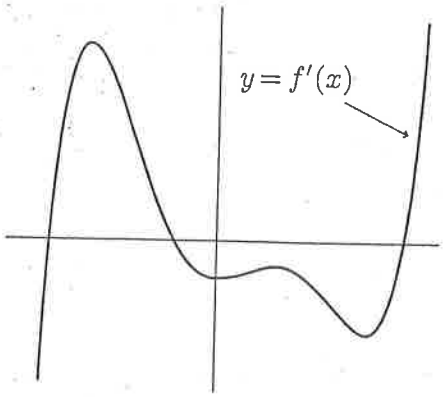
(B) $-\sqrt{\frac{x^3+3}{3}}$

(C) $\sqrt{\frac{x^3+3}{3}}$

(D) $-\sqrt{\frac{2x^3+1}{3}}$

(E) $\sqrt{\frac{2x^3+1}{3}}$

39.



The figure above shows the graph of the derivative of a function f. How many points of inflection does f have in the interval shown?

(A) None

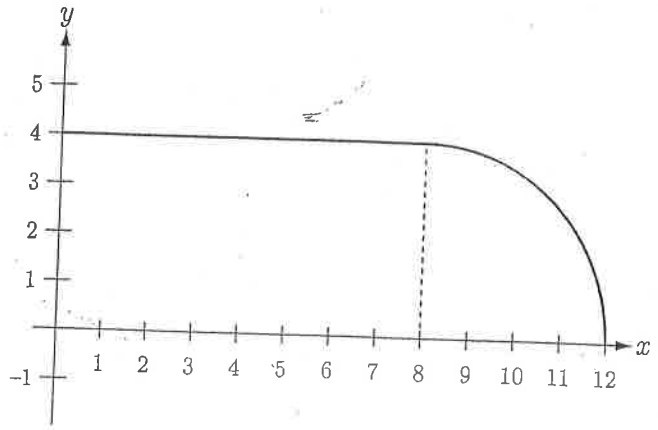
(B) One

(C) Two

(D) Three

(E) Four

40.



As shown in the figure above the function $f(x)$ consists of a line segment from $(0, 4)$ to $(8, 4)$ and one-quarter of a circle with a radius of 4. What is the average (mean) value of this function on the interval $[0, 12]$?

(A) 2

(B) 3.714

(C) 3.9

(D) 22.283

(E) 41.144

42. The amount, $A(t)$, of a certain item produced in a factory is given by

$$A(t) = 4000 + 48(t-3) - 4(t-3)^3$$

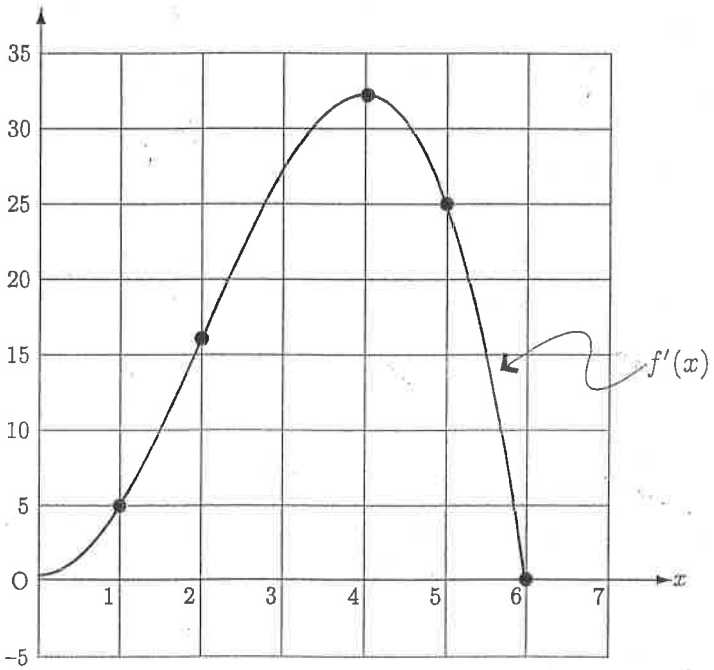
where t is the number of hours of production since the beginning of the workday at 8:00 a.m. At what time is the rate of production increasing most rapidly?

(A) 8:00 a.m.
(B) 10:00 a.m.
(C) 11:00 a.m.
(D) 12:00 noon
(E) 1:00 p.m.

43. The derivative of $4x^2\cos(x)$ is

(A) $-8x\sin(x)$
(B) $8x\cos(x) - 4x^2\sin(x)$
(C) $8x\cos(x) + 4x^2\sin(x)$
(D) $-8x\cos(x) - 4x^2\sin(x)$
(E) $-8x\cos(x) + 4x^2\sin(x)$

45.



Note: This is the graph of $f'(x)$, NOT the graph of $f(x)$.

Let f be a function defined for $0 \le x \le 6$. The graph of $f'(x)$ is shown above. If $f(2) = 10$, which of the following best approximates the maximum value of $f(x)$?

(A) 30
(B) 50
(C) 70
(D) 90
(E) 110